CALCULATION OF VIBRATORY PROCESSES IN LIQUIDS DURING HEAT TRANSFER FROM HOT CHANNEL WALLS

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Vibration modes in liquids during heat transfer with phase transformations are classified here on the basis of generalized test data. Criterial equations of self-excitation and for the natural frequency of harmonic vibrations are derived analytically. These equations are in satisfactory agreement with test results.

The problem of vibrations in liquids during heat transfer from hot channel walls has in recent years aroused considerable interest. These vibrations have a strong effect on the heat transfer and constitute additional mechanical loads on the channel walls.

A method is proposed here for calculating a certain kind of vibratory process.

Basic Test Data. An analysis of available data pertaining to vibrations induced by a supply of heat [1-10] as well as the results of research on this subject conducted by this author jointly with N. A. Kafengauz [11-16] during the 1965-1970 period support the thesis that all vibrations may be tentatively classified, on the basis of their characteristics as well as their excitation mode and probably also their mechanism, into the following three categories: low-frequency vibrations (from a fraction of a cycle per second to tens of cycles per second), high-frequency noise, and high-frequency harmonic vibrations (from hundreds of cycles per second to tens of kilocycles per second).

Vibrations of all three kinds occur in two-phase fluids, low-frequency ones at temperatures of the bulk mass slightly below and high-frequency ones at temperatures of the bulk mass much below the phase-transformation point for the principal component. Low- and high-frequency noise is characterized by a low intensity (pressure amplitudes below 1 to 2 atm. abs) and occurs under subcritical pressures in stationary as well as in moving liquid. High-frequency harmonic vibrations differ from all the other kinds by their characteristics and their excitation mode. Vibrations occur most often at single frequencies, less often they are made up of two or three components and in that case one of them is predominant. The amplitudes of pressure variations reach the order of tens of atmospheres, which makes harmonic pressure fluctuations the most dangerous ones in heat exchangers. Fluctuations of this kind occur only during the flow of a liquid through relatively narrow channels under either subcritical or supercritical pressures; the fluctuation characteristics are the same in both cases.

The subsequent analysis will deal with high-frequency harmonic vibrations only. Among all the possible aspects of this problem, most attention will be paid here to the most important ones: their characteristics and their excitation mode. The study was made with a heat exchanger apparatus described in [14]. The cooling channels were made of thin walled grade Kh18N10T steel tubes 5 to 120 mm long with an inside diameter from 0.8 to 5.6 mm. The tubes were heated by passage of alternating electric current through them. The heat loads could be as high as 40 MW/m^2, with the test pressure ranging from 0.1 to 12 MN/m^2 and the flow velocity ranging from 0 to 50 m/sec. About ten liquids with diverse physical properties were tested, including di-isopropyl-cyclohexane, ethyl alcohol, toluene, isooctane, grade T-1 kerosene and a narrow fraction of keroseneous naphtha with upper boilout points within the 465-540°K range, carbon tetrachloride, water under pressures up to 40 MN/m^2 [17], etc. The test pressures covered both

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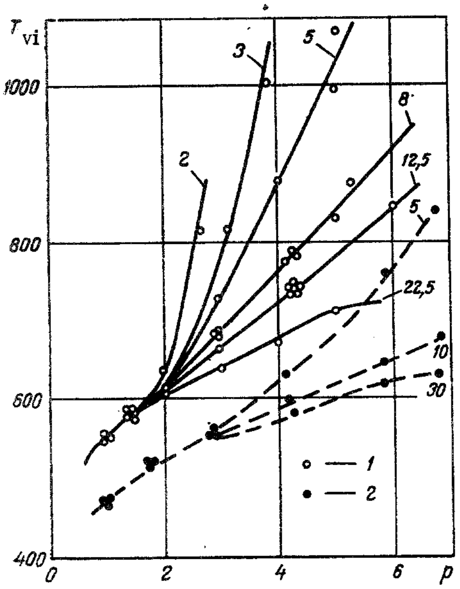

Fig. 1. Temperature at which harmonic vibrations begin T_{vi} (°K), as a function of the pressure p (MN/m^2), for: 1) di-isopropyl-cyclohexane and 2) isooctane; the numbers at the curve indicate the flow velocities in m/sec; tubular channel 1.6 mm in diameter and 30 mm long.

the subcritical and the supercritical ranges for each liquid. The temperature of the liquid at the channel entrance was maintained at the same level of 300°K.

As the parameter characterizing the condition under which the heat transfer enters the fluctuation mode we have adopted the wall temperature, inasmuch as this parameter reveals most clearly the interdependence between the vibration mode and the particular physical properties of a liquid. As the temperature at which vibrations begin T_{vi} must be regarded the wall temperature at which the heat transfer ceases to be convective and, under an increasing heat load on the heat generating element, enters the fluctuation mode; it is the maximum temperature above which the heat transfer cannot be convective.

The temperature at which vibrations begin depends on the modal parameters and on the channel geometry. In Fig. 1 are shown curves representing this temperature as a function of the pressure at constant flow velocities. It is evident, foremost, that the trend of these curves is the same for different liquids. Under subcritical pressures T_{vi} remains independent of the flow velocity: all points fit on a single curve which coincides with the saturation line. Under supercritical pressures the curves depart: the temperature T_{vi} becomes higher at lower flow velocities. It has also been found that in this pressure range T_{vi} is higher in shorter and wider channels [16]. Whether the heat transfer can be excited into a fluctuation mode at all will depend on the flow velocity. A study of this problem has revealed that the minimum velocity at which vibrations are still possible is determined by the pressure. It follows from the data in Fig. 2 that this minimum velocity becomes higher under higher pressure.

The frequency of vibrations increases with increasing pressure (Fig. 3a), with increasing flow velocity (Fig. 3b), and with increasing thermal flux, but it becomes lower in wider or shorter channels, it also decreases as the temperature of the liquid at the channel entrance increases.

On the basis of these data, it is possible not only to assay the mechanism of this phenomenon but also to evaluate the vibrations somewhat more quantitatively.

Mechanism of High-Frequency Harmonic Vibrations. While the wall is heated to the boiling point (under subcritical pressure) or to the temperature of maximum specific heat (under supercritical pressure) of the liquid, a thin layer of the liquid changes into a state with physical properties very different from those of the mainstream liquid.

The layer becomes more compressible and, consequently, capable of wide volume fluctuations. The state of the substance in this layer may, for generality, be called gaseous and the temperature at which the substance changes from liquid to gas may be called the phase-transformation point.

The moving stream carried portions of the gas away from the sites where it has been "generated." Gas "tongues" are formed which extend the gaseous layer beyond the hot channel segment. Such a "tongue" maintains at one end a contact with the gas mass at the hot surface and from here it receives heat together with the mass of gas which has been carried away. At all other sides such a "tongue" borders either on liquid or on the channel wall, both of which are at lower temperatures, and the "tongue" loses heat across these boundaries. When a perturbing pressure is applied, the following happens: the volume of the "tongue" shrinks and its surface area decreases, the heat transfer from the "tongue" to the liquid and to the wall also decreases, but the heat transfer to the "tongue" from the gas mass at the hot surface increases because of the rising temperature of the compressed gas. Therefore, the derivative $d\dot{Q}_{in}/dp$ of thermal flux to a gas "tongue" with respect to pressure is positive and, as we well know, aids the buildup of vibrations. Under the influence of rising pressure in the "tongue" — rising as a result of added heat — the "tongue" begins to expand and displaces adjacent liquid. Owing to the inertia of the liquid, the "tongue" passes through its equilibrium state and will release heat intensively while the pressure in it drops. The resulting rarefaction stops the displacement of liquid and causes it to move toward the wall, so that the "tongue"

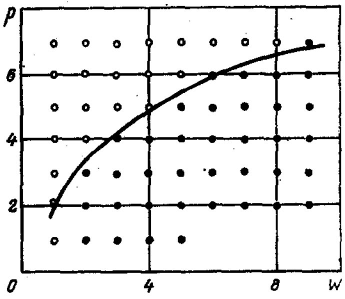

Fig. 2. Stability limit for di-isopropylcyclohexane in p, W (pressure MN/m^2, flow velocity m/sec) coordinates; tubular channel 30 mm long and 1.6 mm in diameter; blank dots represent heat transfer without vibrations, dark dots represent heat transfer accompanied by harmonic vibrations.

becomes again compressed. This process repeats itself. Thus, a gas "tongue" represents a source of vibrations.

The preceding hypothesis is, of course, a very simplified description of the actual process. It offers, however, an explanation of that process and a basis for a quantitative analysis.

As the flow velocity increases, the "tongue" becomes longer and will thus shrink more under a perturbing pressure, and will also receive more thermal energy capable of inducing vibrations. At high flow velocities the temperature of the channel wall does not have to be much above the phase-transformation point. At low flow velocities the "tongue" is short and the derivative $d\dot{Q}_{in}/dp$ can be made as large as necessary by raising the gas temperature and, correspondingly, the wall temperature so that the gas layer becomes sufficiently thick. The "tongue" will be shorter in a shorter channel, because the amount of gas contained in the former is proportional to the amount of gas "generated" at the hot surface. In this case, a higher wall temperature is required for exciting the vibrations. As the channel diameter is increased, the energy of an excitation wave per unit area of channel section becomes smaller so that, consequently, the excitation impulse becomes weaker and a higher wall temperature is required. A pressure rise reduces the compressibility of the gas and this, too, requires more degrees superheat at the channel wall. This analysis of events applies to supercritical pressures. Under subcritical pressures the wall temperature is always near the boiling point of the liquid, inasmuch as the formation of vapor bubbles enhances the heat transfer and thus prevents the wall temperature from rising. It is impossible then to superheat the wall to any significant extent and this explains why, if no harmonic vibrations occur immediately at the start of surface boiling, none will occur later under any additional amount of heat load.

Calculation Formulas. Calculations are made under the following assumptions. The gas at the wall is considered ideal. There is a distinct interphase boundary between gas and liquid; the temperature at this boundary is equal to the phase-transformation point. The thickness and the mean-over-the-mass temperature of the gas layer are linear functions of the layer length. The flow velocity and the physical properties of the gas, namely its density and specific heat, are the same at all sections. The excitation wave has a plane front and a single wave, common to the gas and the liquid, travels through the two-phase stream.

The thickness of the gas layer is much smaller than the channel diameter, the length of the gas "tongue" is much smaller than the length of the gas layer at the hot surface. In view of the small dimensions of the layer, we assume a wave to travel through the gas in an instant of time.

If N_E denotes the power of an excitation wave and N_G denotes the power of gas "tongue" vibrations, then the condition of self-excitation (buildup) can be expressed as

$$N_G > 2N_E. \tag{1}$$

Assuming that both the volume of a gas "tongue" and the heat input to it vary sinusoidally

$$V = V_0 + A_v \sin \omega\tau, \quad \dot{Q}_{in} = A_{\dot{Q}} \sin \omega\tau,$$

we find from the Second Law of Thermodynamics

$$\dot{Q}_{in} d\tau = C_v dT + A p dV,$$

the power of gas "tongue" vibrations to be

$$N_G = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} p dV = \frac{1}{2} \cdot \frac{k-1}{Ak} \cdot \frac{A_p A_{\dot{Q}}}{p}. \tag{2}$$

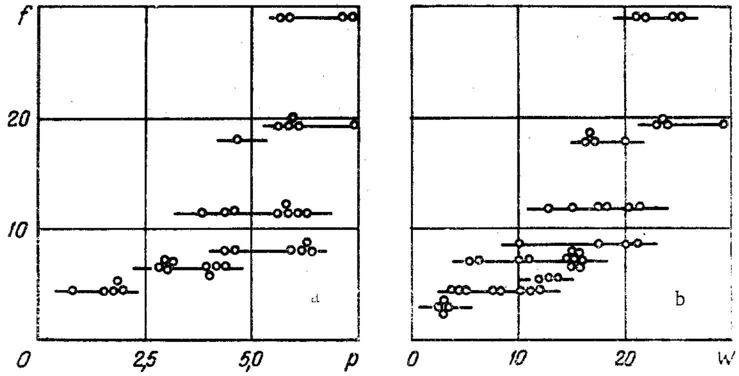

Fig. 3. Frequency of vibrations f (Hz) in di-isopropyl-cyclohexane, as a function of: a) pressure p, MN/m^2 (at a flow velocity W = 20 m/sec); b) flow velocity W, m/sec (under a pressure p = 4.41 MN/m^2); tubular channel 1.6 mm in diameter and 30 mm long.

The power of an excitation wave is found from the relation

$$N_{\mathrm{E}} = F\frac{A_p^2}{(\rho C)_2}\,. \tag{3}$$

A simultaneous solution of Eqs. (1)-(3) yields the dimensionless criterial self-excitation parameter

$$\frac{1}{4}\cdot\frac{k-1}{Ak}\cdot\frac{(\rho C)_2}{pF}\cdot\frac{A_{\dot{Q}}}{A_p} > 1. \tag{4}$$

The initial frequency of vibrations, assuming the perturbations to be infinitesimal, can be determined from the equation of natural vibrations in the two-phase column along a gas "tongue":

$$\rho_2 V_2 \frac{\partial_2 x}{\partial \tau^2} + dpF = 0. \tag{5}$$

The solution to this equation yields

$$\omega^2 = \frac{1}{\beta_2 \rho_2 L_{\mathrm{t}}^2}\,. \tag{6}$$

We next replace the quantities in Eqs. (4) and (6) by quantities stipulated in the original problem: pressure, flow velocity, temperature of the liquid and of the wall, channel length and diameter, and the physical properties of a given liquid under normal conditions.

Let us consider the equation of heat balance for a volume element of gas of length dx.

On the left-hand side, in the direction of flow, an amount of heat equal to

$$\dot{Q}_x = \dot{G}_x C_p T_x \tag{7}$$

enters it together with the mass current.

On the right-hand side an amount of heat equal to

$$\dot{Q}_{x+dx} = \dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x}\,dx \tag{8}$$

leaves it.

From the wall it generally receives an amount of heat equal to

$$d\dot{Q}_{\mathrm{w}} = \pi D q_{\mathrm{w}}\,dx. \tag{9}$$

From the liquid, across the interphase boundary, it receives an amount of heat equal to

$$d\dot{Q}_{\mathrm{L}} = C_p T_{\mathrm{ph}} \frac{\partial \dot{G}_x}{\partial x}\,dx, \tag{10}$$

together with the current of mass undergoing phase transformation.

Part of the heat leaves the gas across the same boundary and returns to the liquid. Using a fictitious thermal conductivity $\bar{\lambda}_x$, which accounts not only for the actual thermal conductivity but also for the convective heat transfer, we can write

$$d\dot{Q}_G = \pi (D - 2h_x) \bar{\lambda}_x \frac{\Delta T_w}{h_x} dx. \tag{11}$$

The equation of heat balance in the given volume element will then be

$$d\dot{Q}_w + d\dot{Q}_L - d\dot{Q}_G - d\dot{Q}_x = 0. \tag{12}$$

Inserting here expressions (7)-(11), we can determine the fictitious thermal conductivity $\lambda_x = \varphi(x)$ and with it, integrating Eq. (12) first over the entire length of the gas layer and then over the "tongue" length L_t only, we can find that "tongue" length:

$$L_t = \frac{1}{2\pi} \cdot \frac{C_p \Delta T_{w,\max} F_G \gamma_G W}{D q_w} \tag{13}$$

as well as the derivative of heat input to the "tongue" with respect to pressure:

$$\frac{d\dot{Q}_{in}}{dp} = \frac{F_G \gamma_G W}{2} C_p \Delta T_{w,\max} [\beta \; (k - 1) + \beta_2]. \tag{14}$$

The resulting expressions can now be inserted into the equations for the vibration frequency and self-excitation (for the latter we assume that $A_{\dot{Q}}/A_p = d\dot{Q}_{in}/dp$).

In order to reduce them to the final form, we will express the properties of a two-phase stream in terms of the properties of both the liquid and the gas, which is rather easy on the basis of the earlier assumption that a single wave travels through both media. The properties of the gas can then be expressed, with some approximation, through the properties of the liquid in terms of referred temperature and pressure, on the basis of the laws of thermodynamic similarity. Concerning the section area covered with gas, we may regard it as proportional to the degrees superheat (thermal overload) of the wall above the gas formation temperature and proportional to the length of the channel segment heated above the phase-transformation point. In turn, according to test data, this length of the hot channel segment is proportional to the length of the heat source element and to the velocity of the liquid. Omitting here all intermediate steps, we show the equations in the final form:

$$\frac{\omega D}{W} = B_1 \left[\frac{\gamma_{Lo}^2 C_{Lo}^2 C_{pLo} T_{cr}}{Agp_{cr}^2} \right]^{n_1} \left(\frac{C_{Lo}}{W} \right)^{n_2} \left(\frac{T_{cr}}{T_L} \right)^{n_3} \left(\frac{D}{L} \right)^{n_4} \left(\frac{p}{p_{cr}} \right)^{n_5}, \tag{15}$$

$$\frac{\Delta T_w}{T_{ph}} = B_2 \left[\frac{Agp_{cr}^2}{\gamma_{Lo}^2 C_{Lo}^2 C_{pLo} T_{cr}} \right]^{m_1} \left(\frac{C_{Lo}}{W} \right)^{m_2} \left(\frac{T_{cr}}{T_L} \right)^{m_3} \left(\frac{D}{L} \right)^{m_4} \left(\frac{p}{p_{cr}} \right)^{m_5}. \tag{16}$$

Coefficients B_1 and B_2 in these equations are constant for any given heat exchanger apparatus.

Not having sufficient information about the physical properties of the liquids tested here and not having sufficiently explored the effect of the temperature of the liquid, it is worthwhile to generalize our test data by first combining the coefficient B_2 (or B_1), the complex group representing the properties of a liquid (expression inside the brackets), the temperature ratio T_{cr}/T_L, and the velocity of sound into a single parameter B_T (or B_ω) with the dimension of velocity and with a value peculiar to each liquid and each heat exchanger.

Equations (15) and (16) will then finally become

$$\frac{\Delta T_w}{T_M} = \left(\frac{p}{p_{cr}} \right)^{2,6} \frac{D}{L} \left(\frac{B_\tau}{W} \right)^2, \tag{17}$$

$$\frac{\omega D}{W} = \left(\frac{p}{p_{cr}} \right)^{0,5} \left(\frac{D}{L} \right)^{0,5} \left(\frac{B_\omega}{W} \right)^2. \tag{18}$$

Parameter B_T is numerically equal to 4.73 for di-isopropyl-cyclohexane, 4.9 for isooctane, 18.5 for grade T-1 kerosene, and 5.65 for the narrow naphtha fraction. The value of parameter B_ω is 2550 for the narrow naphtha fraction and 415 for grade T-1 kerosene.

The form of Eq. (17) corresponds to the supercritical pressure range, for lower pressures the exponent would be different. Using this equation also for subcritical pressures is hardly worthwhile, however: the temperature at which vibrations begin is almost equal then to the boiling point of the liquid, which is known without calculations.

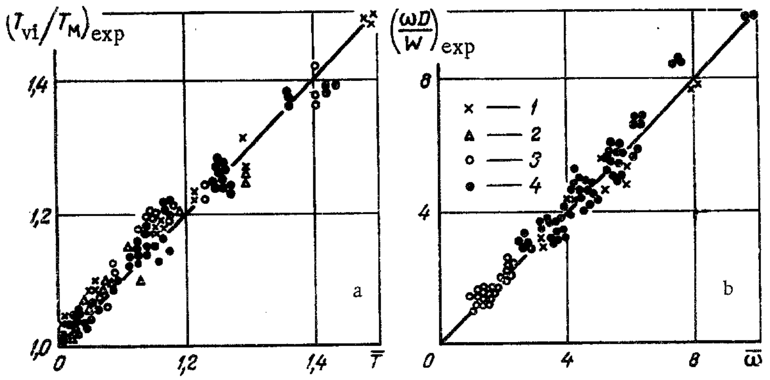

Fig. 4. Correlation between test data and calculated values: a) experimentally and theoretically determined criterial temperature factor for the beginning of vibrations; b) experimentally and theoretically determined criterial parameter for the natural frequency; 1) di-isopropyl-cyclohexane; 2) isooctane; 3) kerosene; 4) naphtha fraction with upper boilout points within 465-540°K; tubular channels 0.8-3.0 mm in diameter and 10-60 mm long; flow velocity W = 2.0-30 m/sec, referred pressure $p/p_{cr} = 1$-3.

The correlation between test data and calculated values is shown in Fig. 4a, b.

In Fig. 4a we have plotted the test values for (T_{vi}/T_M) versus their theoretical values according to the relation

$$\bar{T} = \left(\frac{T_{vi}}{T_M}\right)_{calc} = 1 + \left(\frac{p}{p_{cr}}\right)^{2.6} \frac{D}{L} \left(\frac{B_T}{W}\right)^2,$$

and in Fig. 4b we have plotted the values for

$$\left(\frac{\omega D}{W}\right)_{exp} = \varphi\left[\bar{\omega} = \left(\frac{p}{p_{cr}}\right)^{0,5} \left(\frac{D}{L}\right)^{0.5} \left(\frac{B_\omega}{W}\right)^2\right].$$

The error of calculations does not exceed ±10 to 15%.

The significance of Eq. (17) is not limited to its usefulness for calculating the temperature at which vibrations begin. It can also be used for finding the limits of stable heat transfer in p, W (pressure, flow velocity) coordinates, which would indicate prior to an experiment whether or not vibration is possible in a given apparatus and, in the affirmative case, how the hydrodynamic and other parameters should be changed to avoid it.

Let the channel dimensions as well as the coolant pressure and velocity be known. The maximum wall temperature T_{max} permissible from the standpoint of strength will also be stipulated. Then Eq. (17) can be rewritten as, for example,

$$W_{lim} = \sqrt{B_T^2 \left(\frac{p}{p_{cr}}\right)^{2.6} \frac{D}{L} \cdot \frac{T_M}{T_{max} - T_M}}.$$

Inserting the known and stipulated values into the right-hand side, we obtain the limiting flow velocity W_{lim}. If the given velocity is higher than that, then vibration is possible (of course, at a definite heat load level). If the given velocity is lower than that, on the other hand, then no vibration can occur under any load.

The development of a vibratory process can be inhibited by an increase of pressure, by a wider or shorter channel, or by a decrease of the flow velocity.

The solid line in Fig. 2 represents the stability limit calculated on the basis of 1173°K as the limiting maximum wall temperature (also assumed so in our tests).

The equations derived here are in satisfactory agreement with our test data and, therefore, can serve as a basis for generalizing the data available on harmonic vibrations induced by a heat load as well as a basis for calculating the various vibration modes.

NOTATION

A	is the heat equivalent of mechanical energy;
A_p	is the amplitude of pressure variations;
A_v	is the amplitude of volume variations;
$A_{\dot{Q}}$	is the amplitude of thermal flux variations;
C	is the velocity of sound;
C_p	is the specific heat at constant pressure;
C_v	is the specific heat at constant volume;
D	is the tube diameter;
F	is the area of a tube cross section;
F_G	is the section area covered with gas;
$\dot{G}$	is the flow rate;
h	is the thickness of the gas layer;
g	is the acceleration due to gravity;
k	is the adiabatic exponent;
L	is the length of the heat source element;
L_t	is the length of the gas "tongue";
p	is the pressure;
q_w	is the thermal flux density at the wall;
$\dot{Q}$	is the thermal flux;
$\dot{Q}_{in}$	is the net thermal flux (heat input) to the gas "tongue";
T	is the temperature;
T_{ph}	is the phase-transformation point;
T_M	is the temperature of maximum specific heat;
T_{vi}	is the wall temperature at which vibration begins;
ΔT_w	is the superheat temperature of the wall above the T_{ph} point;
V	is the volume;
W	is the flow velocity;
x	is the displacement;
β	is the adiabatic compressibility;
γ	is the specific gravity;
ρ	is the density;
τ	is the time;
ρC	is the wave resistance;
ω	is the radian frequency.

Subscripts

G	refers to gas;
L	refers to liquid;
cr	refers to critical level;
0	refers to normal conditions;
M	refers to maximum permissible value;
max	refers to largest section at hot end;
calc	refers to calculated value;
exp	refers to test value;
lim	refers to limiting value;
x	refers to a section at distance x;
2	refers to two-phase state;

LITERATURE CITED

1. V. M. Breitman, Heat and Mass Transfer, Vol. 3 [in Russian], Minsk (1965).
2. N. I. Semenov, Teploénergetika, No. 1 (1959).
3. L. N. Proskuryakov, Teploénergetika, No. 2 (1965).
4. M. F. M. Osborne, J. Acoust. Soc. Amer., 19, 21 (1947).
5. M. F. M. Osborne and F. H. Holland, J. Acoust. Soc. Amer., 19, 13 (1947).

6. A. M. Kichigin and L. A. Kesova, Prikl. Mekhan. i Tekh. Fiz., No. 6 (1967).
7. E. I. Nesin, Inzh. Fiz. Zh., 7, No. 9 (1964).
8. E. I. Nesin, Usp. Fiz. Nauk, 87, No. 4 (1965).
9. K. Goldmann, Internatl. Developments in Heat Transfer (1961).
10. W. S. Hines and H. A. Wolf, Roy. Soc. Journal, 32, 3 (1962).
11. N. L. Kafengauz and M. I. Fedorov, Inzh. Fiz. Zh., 11, No. 1 (1966).
12. N. L. Kafengauz and M. I. Fedorov, Teplofiz. Vys. Temp., 5, No. 4 (1967).
13. N. L. Kafengauz and M. I. Fedorov, Oborud. i Sredstva Avtomatiz. v Neftepererabat. i Neftekhim. Promyshl., No. 1 (1968).
14. N. L. Kafengauz and M. I. Fedorov, Teploénergetika, No. 1 (1968).
15. N. L. Kafengauz and M. I. Fedorov, Inzh. Fiz. Zh., 14, No. 5 (1968); 15, No. 3 (1968).
16. N. L. Kafengauz and M. I. Fedorov, Atomnaya Énergiya, 29, No. 4 (1970).
17. L. I. Malkina, G. P. Maksimova, N. L. Kafengauz, and M. I. Fedorov, in: Reports on Sci. Res. at the Krzhizhanovskii ÉNIN [in Russian] (1970).